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Extended Abstract

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Resonant Vibration Control Formats with Equal Modal Damping

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Extended Abstract

The classic resonant control formats are the velocity-velocity format, originally developed by Balas (1979), and the position-position format of Fanson and Caughey (1990). It was later demonstrated that the velocity-velocity format can be recast into acceleration-position form, Sim and Lee (1993). When applied to flexible structures these formats guarantee conditional stability of the position-position format and unconditional stability of the others. The present paper identifies a family consisting of the above formats and their generalized forms, and presents a simple 'optimal calibration' procedure based on 'equal modal damping'.

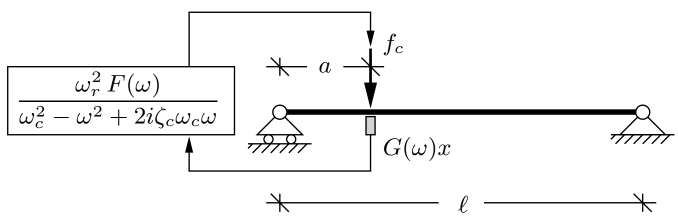

FIGURE 1. Flexible structure with collocated resonant vibration control.

Figure 1 illustrates a structure with an actuator with frequency function $F(\omega)$, controlled by a collocated sensor with frequency function $G(\omega)$. The structural (modal) response $x(t)$ is governed by the equation of motion

$$\ddot{x} + 2\zeta_r\omega_r\dot{x} + \omega_r^2 x \;=\; \omega_r^2 F(\omega)\xi + \omega_r^2 f/k_r. \tag{1}$$

with resonant angular frequency ω_r and (modal) damping ratio ζ_r. The resonant controller signal $\xi(t)$ is governed by the second order equation

$$\ddot{\xi} + 2\zeta_c\omega_c\dot{\xi} + \omega_c^2\xi \;=\; \omega_r^2 G(\omega)x. \tag{2}$$

with controller frequency ω_c and equivalent controller damping ratio ζ_c. The various sensor/actuator frequency function combinations are shown in Table 1, where velocity-velocity is represented via its equivalent acceleration-position form. It is seen that the extended formats are obtained by adding a velocity term to the position term in the original format. This particular form of the extension permits a simple generalization of the stability results for MDOF systems. The effect of the modification of the simple formats is to introduce an extra factor ω in the frequency response function of the controlled structure as illustrated in Figs. 2 and 3.

TABLE 1. Frequency functions for resonant control formats.

	Acceleration	Extended Acceleration	Position	Extended position
$G(\omega)$	ω^2	ω^2	ω_r^2	ω_r^2
$F(\omega)$	ω_c^2	$\omega_c^2 + 2i\zeta_c\omega_c\omega$	ω_c^2	$\omega_c^2 + 2i\zeta_c\omega_c\omega$

Robust and tractable properties are obtained by imposing the condition that the two modes generated via introduction of the resonant controller should have identical modal damping ratio. The origin of this

Table 2. Resonant control parameters.

	Acceleration	Extended Acceleration	Position	Extended position
ω_c	ω_r	$\omega_r/(1+\alpha)$	$\omega_r/\sqrt{1-\alpha}$	$\omega_r\sqrt{1-\alpha}$
ζ_c	$\sqrt{\frac{1}{2}\alpha}$	$\sqrt{\frac{1}{2}\alpha/(1+\alpha)}$	$\sqrt{\frac{1}{2}\alpha}$	$\sqrt{\frac{1}{2}\alpha/(1-\alpha)}$

idea is the observation that this is an alternative approach to the classic design problem of the 'tuned mass absorber', Krenk (2005). This 'equal modal damping' property determines the controller frequency ω_c. The controller damping parameter ζ_c is then determined to produce a suitable separation between the modal frequencies. Also this property can be represented by the pole locations alone. When a gain parameter α is introduced to give the product of the frequency response functions the form $\alpha F(\omega)G(\omega)$, the corresponding parameters ω_c and ζ_c are given in Table 2. Acceleration and position control and their extended forms are illustrated in Figs. 2 and 3. The plateau around resonance is indicated by the dashed curves $\propto \omega^{-n}$ with $n = 1, 0$ and $n = 2, 1$ for acceleration and position feedback, respectively.

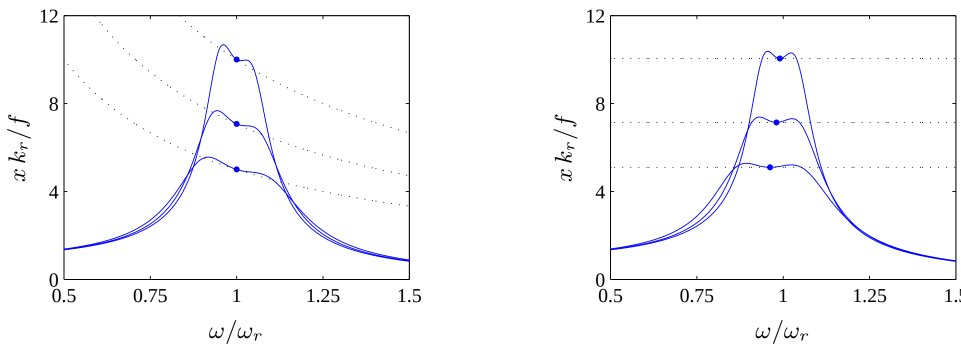

Figure 2. Amplitude for acceleration feedback: (a) simple, (b) extended, $\alpha = 0.02,\ 0.04,\ 0.08$.

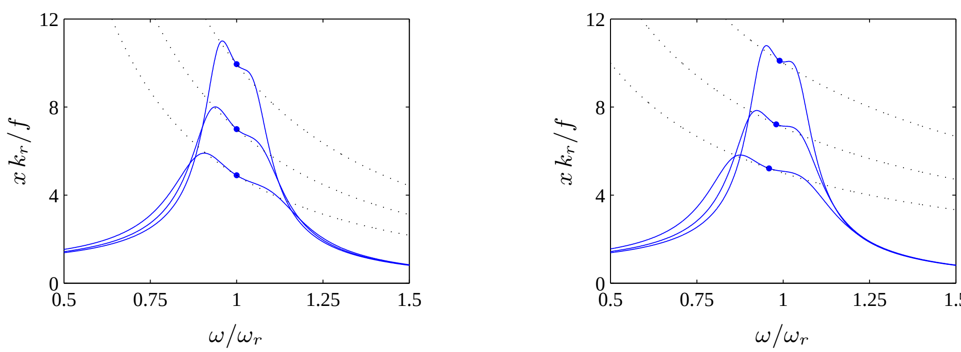

Figure 3. Amplitude for position feedback: (a) simple, (b) extended, $\alpha = 0.02,\ 0.04$ and 0.08.

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